

Sunspot Volatility and the Price of Peace

Robert J. Carroll

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Abstract

The costs of maintaining peace under anarchy can make an otherwise risk-averse state prefer volatility. In a model with two countries and an outside investor, the countries play a war game in which a strong country may pay tribute to deter a weaker adversary. The tribute distorts the strong country's reduced-form payoff over wealth, creating a convex kink at the boundary where the country switches from paying tribute to fighting. Over this range, the country would prefer a gamble—slightly poorer in some states of the world (where tribute is cheaper) and slightly richer in others (where it fights on better terms)—to the certainty of an intermediate endowment. When the country can trade state-contingent claims with the outside investor, a sunspot equilibrium exists: an extrinsic random variable, unrelated to fundamentals, determines the terms of engagement between the two countries. The mechanism requires no private information, commitment problems, or risk-acceptant preferences. The non-concavity arises endogenously from the structure of international bargaining.

Keywords: war, risk acceptance, volatility, conflict, tribute, sunspot equilibrium

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Peace is not free. When two countries interact under anarchy, avoiding war requires costly investments in deterrence—standing armies, military alliances, side payments to potential aggressors—that consume resources both sides would prefer to spend elsewhere. This observation is central to a large literature on the costs of anarchy [Fearon, 2018, Powell, 1993]. Coe [2011] argues that costly peace—encompassing arming, imposition by outside actors, and predation through tribute—is a neglected rationalist explanation for war, on par with private information and commitment problems. The transfers required to maintain peace are not incidental: they are a structural feature of international bargaining, growing with the asymmetry between states.

This paper focuses on one form of these costs—side payments (tribute) from a stronger country to a weaker one to buy off the threat of war—and shows that they can do more than waste resources: they can distort the effective risk preferences of states, creating a demand for volatility. When a risk-averse country must pay tribute to buy off a potential aggressor, the tribute distorts its reduced-form payoff over wealth: the payoff tracks the utility function in the region where peace is free, then sags below it where tribute is required. The result is a payoff that is not globally concave. Over a range of endowments, a fundamentally risk-averse country would prefer a gamble over its wealth to the certainty of an intermediate endowment that attracts aggression.

If the country can access a financial market—trading state-contingent claims with an outside party—it can split its endowment across states of the world: poorer in some (where tribute is cheaper) and richer in others (where it fights on better terms). The lottery strictly improves its expected payoff. An extrinsic random variable, entirely unrelated to fundamentals—a sunspot—can thereby determine whether the country pays tribute or goes to war. For parameters where the non-concavity is strong, the sunspot can determine genuine peace or conflict.

The paper builds a three-actor model to formalize this argument. Two countries, A and B , interact in a war game with endogenous effort, transfers, and a Tullock contest; this game is standard in the conflict literature [*e.g.*, Beviá and Corchón, 2010]. A third party, W (the “World”), does not participate in the war game but can trade Arrow securities with A on a sunspot variable. The model is solved in two stages. First, the equilibrium of the war game is characterized as a function of A ’s resources, showing that A ’s reduced-form payoff is continuous, monotone, and not concave. Second,

when the payoff is not concave, there exists a nontrivial sunspot equilibrium in which A and W trade, the sunspot determines whether A enters the war game rich or poor, and peace or conflict follows accordingly.

The mechanism does not require private information, commitment problems, or behavioral departures from rationality—the three channels most commonly invoked to explain inefficient conflict [Fearon, 1995, Powell, 2006]. All actors have complete information and standard preferences. The non-concavity arises endogenously from the structure of international bargaining: the tribute that a strong country must pay to deter a weaker adversary is the source of the distortion. In this sense, the costs of anarchy identified by Fearon [2018] are not merely wasteful—they are distortionary, reshaping the effective risk preferences of states in ways that generate demand for volatility.

Related literature. The paper connects two largely separate literatures: the study of costly peace in international relations and the theory of sunspot equilibria in economics.

On the conflict side, Fearon [1995] frames the central puzzle: war is costly, so rational states should always find a bargain both prefer. His three rationalist explanations—private information, commitment problems, and issue indivisibilities—have structured the field for three decades. Powell [2006] argues that indivisibilities reduce to commitment problems, and that risk-acceptant preferences do not explain war because costless lotteries exist. The mechanism here is distinct: the non-concavity is not a property of the primitive utility function (which is concave) but of the reduced-form payoff induced by strategic interaction. The “lottery” that resolves the non-concavity is not hypothetical—it is implemented through a financial market with a willing counterparty.

The model can be read as asking what happens when Fearon’s costs of peace [Fearon, 2018]—here taking the form of tribute rather than arms—are combined with risk aversion. Garfinkel [1990] shows that military spending fluctuates endogenously with economic activity, an early demonstration that peace costs generate volatility; but her volatility is intrinsic, driven by endowment shocks, whereas the volatility here is extrinsic. Coe [2011] models arming, imposition, and predation as distinct sources of costly peace and applies each to a historical case; Meiorowitz et al. [2024] extend the logic to repeated bargaining. Neither connects the costs of peace to financial markets or sunspot equilibria. Separately, a growing empirical literature finds

that financial liberalization can destabilize: [Gartzke \[2007\]](#) argues that capital market integration promotes peace, while [Bussmann and Schneider \[2007\]](#) and [Chapman and Reinhardt \[2013\]](#) find that the *process* of opening to international capital increases conflict risk. The present paper provides a formal mechanism consistent with both findings (Section 3).

The closest comparison in the conflict literature is [Goemans and Fey \[2009\]](#), who show that domestic political institutions can create non-concave (S-shaped) payoffs for leaders, inducing rational risk-seeking in military strategy. The mechanism here differs in source and scope. Their non-concavity comes from the selectorate: leaders who lose a little are punished as severely as those who lose a lot. The non-concavity in this paper comes from international bargaining itself—the tribute required to buy peace—and requires no domestic institutional structure. Their gambling occurs within the conflict (risky military strategies); here it occurs before the conflict, in a financial market, and can generate peace in one state and conflict in another.

On the economics side, [Prescott and Shell \[2002\]](#) establish that when agents’ reduced-form payoffs are not concave, they demand randomization, and extrinsic uncertainty (sunspots) can serve as the randomization device. This paper applies that logic in a conflict setting: the war game generates the non-concavity, and the Arrow security market supplies the randomization. The foundational theory of sunspot equilibria [[Cass and Shell, 1983](#), [Shell, 1977](#)] shows that extrinsic uncertainty can matter when market participation is restricted or markets are incomplete. The model does feature restricted participation—country B does not trade—but the driving mechanism is the non-concavity of A ’s reduced-form payoff, not the market restriction.

The paper proceeds as follows. Section 1 describes the model. Section 2 solves the war game and the futures market, establishing the main results. Section 3 connects the mechanism to the empirical literature on financial integration and conflict. Section 4 concludes.

1 Model

There are three actors: two countries A and B , and an outside investor W (which we will call the World). The interaction unfolds in two stages. In the first stage, A and W trade Arrow securities over a payoff-irrelevant sunspot variable. The second stage takes place after this variable has been realized; in it, A and B play a war game (itself in a series of substages) in which W

does not participate. The main question is whether the sunspot variable can affect the outcome of this second-stage interaction.

All three actors are concerned with consumption of a single homogenous good. Actor $j \in \{A, B, W\}$ enters the interaction with an endowment $\omega_j > 0$ of this good. The two countries share a common constant relative risk aversion (CRRA) utility function

$$u(x) = x^\alpha, \quad \text{where } \alpha \in (0, 1]. \quad (1)$$

When $\alpha = 1$, the countries are risk-neutral; when $\alpha < 1$, they are risk-averse. Throughout, we write u rather than x^α when no confusion arises. Upon inspection we see that u is strictly increasing, concave, twice continuously differentiable on $(0, \infty)$, and satisfies $u(0) = 0$.¹ The World has a utility function $u_W : \mathbb{R}_+ \rightarrow \mathbb{R}$, which is strictly increasing, strictly concave, and twice continuously differentiable on $(0, \infty)$, but otherwise unrestricted.

On ex ante risk aversion. The concavity of u and u_W ensures that all actors are risk averse entering into the interaction.

1.1 First stage: futures market

The sunspot variable $\theta \in \{0, 1\}$ provides extrinsic uncertainty that can be traded on in the first stage. The probability of sunspots ($\theta = 1$) is $\pi \in (0, 1)$; this probability is common knowledge. Prior to its realization, A and W can trade Arrow securities that pay off in the second stage. A claim on state θ pays 1 unit of the good if the realized state is θ and nothing otherwise.

Formally, each trader $j \in \{A, W\}$ chooses a portfolio $r_j = (r_j(0), r_j(1))$ of net trades in Arrow securities, where $r_j(\theta)$ is the net purchase of state- θ claims. The price of the state-1 claim is $q \in (0, 1)$, and the price of the state-0 claim is $1 - q$.² Each trader faces the budget constraint

$$(1 - q)r_j(0) + qr_j(1) \leq 0, \quad (\text{BC}_j)$$

¹The mechanism identified here does not depend on the CRRA functional form. However, in many of the calculations it yields clean closed forms and makes the structure more transparent. The key qualitative results hold for general concave u .

²These prices sum to one, so that a bundle of one unit of each claim replicates a sure unit of the good; this means the good is numeraire.

so that net purchases of one security must be offset by net sales of the other. The market clearing condition is

$$r_A(\theta) + r_W(\theta) = 0 \quad \text{for both } \theta \in \{0, 1\}. \quad (\text{MC})$$

Country B does not trade: $r_B(\theta) = 0$ for both $\theta \in \{0, 1\}$.

We add the requirement that the traders' portfolios be *feasible*, meaning that they cannot lead to negative consumption in any state. Formally, this means that for both $j \in \{A, W\}$ and both $\theta \in \{0, 1\}$, we have $\omega_j + r_j(\theta) \geq 0$.

After the portfolios are chosen, the sunspot variable θ is realized, and the payoffs from the futures market are delivered. This ends the World's involvement in the interaction; it consumes its payoff and exits. Thus, it chooses its portfolio to maximize the expected utility

$$(1 - \pi)u_W(\omega_W + r_W(0)) + \pi u_W(\omega_W + r_W(1)) \quad (2)$$

subject to the budget and feasibility constraints. As for country A : the subsequent war game has a unique equilibrium (as we will see in the sequel), so we may write $\Pi_A(V_A)$ for A 's reduced-form equilibrium payoff as a function of their resources with the parameters of the war game held fixed. Country A chooses its portfolio r_A to maximize

$$(1 - \pi)\Pi_A(\omega_A + r_A(0)) + \pi\Pi_A(\omega_A + r_A(1)) \quad (3)$$

again subject to the budget and feasibility constraints.

On the market for insurance. If Π_A were concave, the Arrow security market would function as insurance: A would smooth consumption across states, and the sunspot would be irrelevant. (Two risk averse parties cannot both want to trade in an otherwise-closed futures market.) The question is whether the war game can distort Π_A enough to reverse this logic.

1.2 Second stage: war contest

After the sunspot variable is realized, the two countries play a war game as developed by [Beviá and Corchón \[2010\]](#). Country A begins with $V_A = \omega_A + r_A(\theta)$, and B with $V_B = \omega_B$. When we wish to emphasize the total resources, we write Ω , where $\Omega = V_A + \omega_B$.

The game has three substages.

1. *Transfer.* One of the countries may transfer resources to the other. Call the post-transfer endowments \hat{V}_A and \hat{V}_B , and impose the lump-sum requirement that $\hat{V}_A + \hat{V}_B = \Omega$.³
2. *Declaration.* Each country simultaneously decides whether to declare war. If neither declares war, then the game ends and payoffs are given by $u(\hat{V}_A)$ and $u(\hat{V}_B)$.
3. *Contest.* If either country declares war, then the countries simultaneously choose effort levels $e_i \in [0, \hat{V}_i]$. Country i wins the contest with probability

$$p_i(e_A, e_B) = \begin{cases} \frac{e_i}{e_A + e_B}, & \text{if } e_A + e_B > 0, \\ 1/2, & \text{otherwise.} \end{cases} \quad (4)$$

Effort is chosen to maximize the expected utility

$$U_i(e_A, e_B) = p_i(e_A, e_B)u(\Omega - k(e_A + e_B)),$$

where $k \in (0, 1]$ captures how much the winner can recuperate effort.

An equilibrium of the second-stage game is a pair of effort levels, a pair of declarations, and an appropriately-designed transfer decision. An equilibrium of the entire interaction is an equilibrium of the second-stage game and a pair of portfolio choices that clear markets with utility maximization as described above.

On the sunspot. Sunspots do not matter with respect to the fundamentals of the interaction. θ does not influence the consumption utility functions, nor the endowments, nor the marginal cost of war. We are interested in whether it can influence the outcome nonetheless.

2 Analysis

The game is solved backwards.

³The terms of the transfer—*i.e.*, who transfers to whom, and to achieve what strategic purpose—are left open for now, as they depend on where we are in the space.

2.1 Stage two

We normalize $\omega_B = 1$ throughout.⁴ Write $\Omega = V_A + 1$ for total resources, where $V_A = \omega_A + r_A(\theta)$ is A 's endowment after the futures market payout.

Standard results give us that (1) the war contest has a pure-strategy Nash equilibrium; (2) this equilibrium is unique; and (3) in any equilibrium, $e_j^* > 0$ for both countries. We denote the equilibrium by (e_A^*, e_B^*) and country i 's equilibrium war payoff by

$$\Psi_i = \frac{e_i^*}{e_A^* + e_B^*} \cdot u(\Omega - k(e_A^* + e_B^*)). \quad (5)$$

The interior (unconstrained) equilibrium effort level is

$$e^\dagger = \frac{V_A + 1}{2(1 + \alpha)k}, \quad (6)$$

which reduces to $\Omega/4k$ when $\alpha = 1$ [Beviá and Corchón, 2010].

2.1.1 Symmetric regimes

When both countries can afford e^\dagger ($e^\dagger \leq \min\{V_A, 1\}$), the equilibrium is (e^\dagger, e^\dagger) . When both are constrained ($e^\dagger \geq \max\{V_A, 1\}$), it is $(V_A, 1)$. In both cases, neither country prefers war, so $\Pi_A(V_A) = V_A^\alpha$ (Appendix A.1).

2.1.2 The asymmetric regime

The regime of interest is $1 < e^\dagger < V_A$, so that B is budget-constrained but A is not. The condition $e^\dagger > 1$ is equivalent to $V_A > \bar{V}$, where

$$\bar{V} \equiv 2(1 + \alpha)k - 1. \quad (7)$$

In this regime, $e_B^* = 1$ and A 's first-order condition determines total effort $s^* = e_A^* + 1$. Writing $Z = V_A + 1 - k s$ for the post-contest surplus, the FOC reduces to a quadratic in s whose positive root is

$$s^* = \frac{-(1 - \alpha)k + \sqrt{(1 - \alpha)^2 k^2 + 4\alpha k (V_A + 1)}}{2\alpha k}. \quad (8)$$

(The derivation is in Appendix A.2.) When $\alpha = 1$, this reduces to $s^* = \sqrt{(V_A + 1)/k}$, recovering Beviá and Corchón [2010]'s equation (3.4).

⁴Since B does not participate in the futures market, its endowment ω_B is a fixed parameter that merely sets the scale. Setting $\omega_B = 1$ is therefore without loss of generality; all formulas extend to general $\omega_B > 0$ by replacing 1 with ω_B .

A prefers peace. Country A (the richer, unconstrained country) always prefers peace to war without transfers: $\Psi_A < u(V_A)$. The argument uses concavity of u and the fact that $e_A^* < V_A$ (Appendix A.2).

B 's war condition. Country B prefers war iff

$$\Psi_B = \frac{1}{s^*} u(Z^*) > u(1), \quad (9)$$

which, since $u(1) = 1$, simplifies to

$$Z^{*\alpha} > s^*. \quad (10)$$

Define V_A^{war} as the value of V_A at which equality holds. For $V_A > V_A^{\text{war}}$, country B prefers war; for $\bar{V} < V_A < V_A^{\text{war}}$, B prefers peace despite being constrained.

When $\alpha = 1$, the condition reduces to $V_A + 1 > (1 + k)^2/k$, recovering Beviá and Corchón [2010]'s threshold $V_2/V < k/(1+k)^2$.

Tribute. When B prefers war, A can offer a transfer $T \geq 0$ to buy peace. After the transfer, the contest is fought at $(V_A - T, 1 + T)$ with total resources Ω unchanged. The smallest transfer that makes B willing to accept peace satisfies

$$u(1 + \bar{T}) = \Psi_B(\bar{T}), \quad (11)$$

where $\Psi_B(\bar{T})$ is B 's war payoff at the post-transfer endowments (computed from (8) with $1 + T$ in place of 1). Country A pays tribute iff peace at cost \bar{T} beats war at the original endowments:

$$u(V_A - \bar{T}) \geq \Psi_A(0). \quad (12)$$

Define V_A^{fight} as the value of V_A at which equality holds in (12). For $V_A^{\text{war}} < V_A \leq V_A^{\text{fight}}$, A pays tribute; for $V_A > V_A^{\text{fight}}$, war occurs.

2.1.3 The reverse asymmetry

When A is constrained but B is not ($V_A < e^\dagger < 1$), the roles reverse: A is “all in” and B plays a best response. In this regime, $\Pi_A(V_A) \geq V_A^\alpha$ —either B pays

tribute to A , or A fights a favorable contest.⁵ Define \underline{V} as the supremum of V_A values at which the reverse asymmetry yields $\Pi_A(V_A) > V_A^\alpha$, with $\underline{V} = 0$ if no such values exist.

2.1.4 Summary of stage two

Three features of the stage-two equilibrium deserve emphasis. First, when countries are sufficiently equal—either both rich enough to be unconstrained, or both poor enough to be fully constrained—peace is costless. The war game is a pure deadweight loss, and both sides know it. Second, war becomes possible only when one country is constrained and the other is not. The constrained country is “all in”: it wagers everything for a share of a prize that far exceeds its endowment, making war attractive to the poor and tribute expensive for the rich. Third, whether the rich country pays tribute or fights depends on how lopsided the endowments are. When inequality is moderate, the required tribute is small and peace is cheap; when inequality is extreme, fighting is preferable to paying. The result is that war occurs precisely in the corners of the endowment space where one country is very rich and the other very poor.

Collecting the results, A ’s reduced-form payoff as a function of V_A (holding $\omega_B = 1$ fixed) is:

$$\Pi_A(V_A) = \begin{cases} V_A^\alpha & \text{if } \underline{V} \leq V_A \leq V_A^{\text{war}}, \\ (V_A - \bar{T}(V_A))^\alpha & \text{if } V_A^{\text{war}} < V_A \leq V_A^{\text{fight}}, \\ \frac{e_A^*(V_A)}{s^*(V_A)} \cdot Z^*(V_A)^\alpha & \text{if } V_A > V_A^{\text{fight}}. \end{cases} \quad (13)$$

For $V_A < \underline{V}$, $\Pi_A(V_A) \geq V_A^\alpha$ (the reverse asymmetry): A either is left unbothered or is able to successfully demand tribute from B .

The non-concavity. For $V_A \in [\underline{V}, V_A^{\text{war}}]$, $\Pi_A(V_A) = V_A^\alpha$, which is strictly concave. For $V_A > V_A^{\text{war}}$, $\Pi_A(V_A) < V_A^\alpha$: A always prefers free peace to war (proved above), so any tribute or contest cost pushes Π_A below $u(V_A)$. The gap never closes—even in the war region, $\Psi_A(0) < u(V_A)$ —but as $V_A \rightarrow \infty$, A becomes so dominant that $\Psi_A(0)/u(V_A) \rightarrow 1$.

⁵This region is bounded away from V_A^{war} and plays no role in the non-concavity or sunspot arguments that follow.

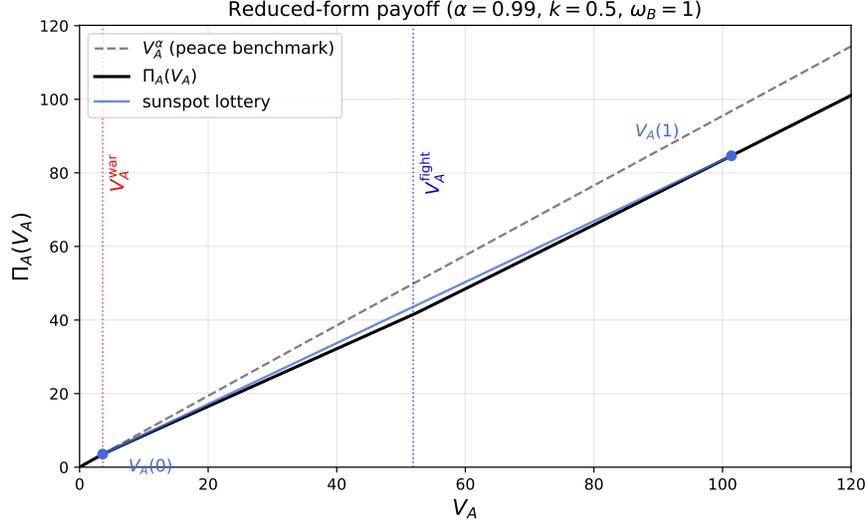


Figure 1: Country A 's reduced-form payoff $\Pi_A(V_A)$ (solid) versus the peace benchmark V_A^α (dashed), for $\alpha = 0.99$, $k = 0.5$, $\omega_B = 1$. At $V_A^{\text{war}} \approx 3.6$, country B begins to prefer war; A pays tribute and Π_A drops below V_A^α . At $V_A^{\text{fight}} \approx 51.9$, tribute becomes too expensive and war occurs. The chord connects the endpoints of a computed sunspot equilibrium ($\pi = 0.1$, $q^* \approx 0.102$): $V_A(0) \approx 3.6$ (peace) and $V_A(1) \approx 101.4$ (conflict). The chord lies above Π_A , confirming that A gains from the lottery.

The result is that Π_A departs downward from V_A^α at $V_A = V_A^{\text{war}}$ and never returns (Figure 1). This is a non-concavity: there exist $V_A^L < V_A^{\text{war}} < V_A^R$ and weights $\lambda \in (0, 1)$ such that

$$\lambda \Pi_A(V_A^L) + (1 - \lambda) \Pi_A(V_A^R) > \Pi_A(\lambda V_A^L + (1 - \lambda) V_A^R). \quad (14)$$

In words: country A would prefer a lottery over a low endowment V_A^L (where peace is free) and a high endowment V_A^R (where war is cheap to win) to the certainty of the average—where A is rich enough to attract B 's aggression but not rich enough to fight cheaply.

We may now state the main result of this section.

Proposition 1. *Fix $k \in (0, 1]$ and $\alpha \in (0, 1)$. Country A 's reduced-form payoff $\Pi_A(V_A)$ satisfies:*

1. (Peace region.) $\Pi_A(V_A) = V_A^\alpha$ for all $V_A \in [\underline{V}, V_A^{\text{war}}]$.

2. (Costs of peace.) $\Pi_A(V_A) < V_A^\alpha$ for all $V_A > V_A^{war}$.
3. (Asymptotic dominance.) As $V_A \rightarrow \infty$, total effort $s^* = O(\sqrt{V_A})$, so the cost of war becomes negligible relative to the prize: $\Pi_A(V_A)/V_A^\alpha \rightarrow 1$.⁶
4. (Continuity.) Π_A is continuous on $(0, \infty)$.
5. (Non-concavity.) Π_A has a convex kink at V_A^{fight} : the right derivative strictly exceeds the left derivative,

$$\Pi'_A(V_A^{fight+}) > \Pi'_A(V_A^{fight-}). \quad (15)$$

In particular, Π_A is not concave on $(0, \infty)$.

The non-concavity of Π_A has a striking implication. When a decision-maker's payoff is concave in wealth, uncertainty is costly: the agent would always prefer a sure endowment to a gamble with the same mean. But when the payoff is *not* concave, the opposite can happen—the agent may strictly prefer the gamble.

The convex kink at V_A^{fight} captures a sharp discontinuity in the marginal return to wealth. In the tribute region (just below V_A^{fight}), an extra dollar of wealth is partially captured by B : tribute rises at rate $\bar{T}' > 0$, so A 's effective wealth grows at only rate $1 - \bar{T}'$. In the war region (just above V_A^{fight}), there is no tribute; an extra dollar directly improves A 's position in the contest. This jump in marginal returns means that A at V_A^{fight} would benefit from randomizing its endowment: slightly poorer in some states (paying less tribute) and slightly richer in others (fighting on better terms).

This is where the outside investor W enters. If there is a market in which A can trade claims contingent on an extrinsic random variable—the sunspot θ —then A can rearrange its endowment across states. In state $\theta = 0$, A gives up some resources, entering the tribute region with a lower $V_A(0)$. In state $\theta = 1$, A receives resources, crossing into the war region with $V_A(1) > V_A^{fight}$. The convex kink ensures that the chord connecting these two points lies above Π_A at ω_A (Figure 1), so the rearrangement raises A 's expected payoff.⁷

⁶From (8), s^* is the positive root of a quadratic with leading coefficient αk and constant term $V_A + 1$, so $s^* \sim \sqrt{(V_A + 1)/(\alpha k)}$. Note also that Π_A is differentiable everywhere except at the two kinks V_A^{war} and V_A^{fight} .

⁷For α close to 1, the non-concavity is strong enough to support a more dramatic sunspot equilibrium: A randomizes between a low endowment below V_A^{war} (genuine peace, no tribute) and a high endowment above V_A^{fight} (war). See the numerical examples below.

But who takes the other side of the trade? The World, W , has a strictly concave utility function u_W . For W , uncertainty is purely costly— W has no war game to play, no tribute to pay, and no reason to want volatility. So when A wants to *buy* exposure to the sunspot (high V_A in one state, low in the other), W is willing to *sell* it, provided the price is right. The gains from trade are mutual: A benefits from the non-concavity, and W is compensated for bearing risk it dislikes.

The question is whether a price exists at which both sides want to trade. If so, the sunspot—a variable with no direct effect on fundamentals—affects the allocation of resources across states, and thereby affects the outcome of the war game. We now formalize this.

2.2 Stage one

We now solve the futures market described in Section 1, taking as given the reduced-form payoff Π_A from Proposition 1. A *sunspot equilibrium* is a price q^* and portfolios (r_A^*, r_W^*) satisfying budget constraints (BC _{j}), market clearing (MC), feasibility, and individual optimality. The no-trade allocation $r_A = r_W = 0$ is always an equilibrium: at any price q , if one trader does not trade, the other has no counterparty. A *nontrivial* sunspot equilibrium is one in which $r_A^* \neq 0$ —so that V_A differs across states and the sunspot affects the war game.

2.2.1 Reducing to one variable

By market clearing, $r_W(\theta) = -r_A(\theta)$ for each θ , so the entire allocation is determined by A 's portfolio. Moreover, the budget constraint $(1 - q)r_A(0) + qr_A(1) \leq 0$ binds at any optimum where trade occurs (neither trader leaves money on the table), so

$$r_A(0) = -\frac{q}{1 - q} r_A(1). \quad (16)$$

Let $\varepsilon \equiv r_A(1)$ denote A 's net purchase of the state-1 claim. Then $r_A(0) = -q\varepsilon/(1 - q)$, and the post-trade resources are

$$V_A(1) = \omega_A + \varepsilon, \quad V_A(0) = \omega_A - \frac{q}{1 - q} \varepsilon, \quad (17)$$

$$\omega_W + r_W(1) = \omega_W - \varepsilon, \quad \omega_W + r_W(0) = \omega_W + \frac{q}{1 - q} \varepsilon. \quad (18)$$

The entire problem reduces to finding a price $q \in (0, 1)$ and a trade size ε such that both A and W are optimizing.

2.2.2 Optimality conditions

The investor W has a strictly concave objective, so W 's optimum is characterized by the first-order condition. Differentiating with respect to ε (using $r_W(1) = -\varepsilon$ and $r_W(0) = q\varepsilon/(1 - q)$):

$$\pi u'_W(\omega_W - \varepsilon) = \frac{q}{1 - q} (1 - \pi) u'_W\left(\omega_W + \frac{q}{1 - q} \varepsilon\right). \quad (19)$$

At $\varepsilon = 0$ (no trade), both arguments equal ω_W , and the condition reduces to $q = \pi$. That is, W is content with no trade if and only if the price of the state-1 claim equals the probability of state 1.

Country A maximizes $(1 - \pi) \Pi_A(V_A(0)) + \pi \Pi_A(V_A(1))$. Since Π_A is differentiable everywhere except at the kinks V_A^{war} and V_A^{fight} (Proposition 1), the first-order condition in ε is

$$\pi \Pi'_A(V_A(1)) = \frac{q}{1 - q} (1 - \pi) \Pi'_A(V_A(0)). \quad (20)$$

At $\varepsilon = 0$, this reduces to $q = \pi$ —the same condition as W 's. So at price $q = \pi$, both agents are satisfied with no trade, confirming that the trivial equilibrium exists.

The question is whether A *wants* to deviate from $\varepsilon = 0$ at price $q = \pi$ —and if so, whether there is a nearby price at which both agents agree on a nonzero ε .

2.2.3 A's gain from trade

Write A 's expected payoff as a function of ε at price q :

$$F(\varepsilon; q) = (1 - \pi) \Pi_A\left(\omega_A - \frac{q}{1 - q} \varepsilon\right) + \pi \Pi_A(\omega_A + \varepsilon). \quad (21)$$

Suppose $\omega_A > V_A^{\text{war}}$, so that $\Pi_A(\omega_A) < \omega_A^\alpha$ by Proposition 1. At $q = \pi$ (actuarially fair pricing), A can choose $\varepsilon > 0$ to place $V_A(0) = \omega_A - \frac{\pi}{1 - \pi} \varepsilon$ below V_A^{war} —where peace is free and $\Pi_A = V_A^\alpha$ —and $V_A(1) = \omega_A + \varepsilon$ above it. Whether this improves A 's expected payoff, i.e., whether

$$F(\varepsilon; \pi) = (1 - \pi) \Pi_A(V_A(0)) + \pi \Pi_A(V_A(1)) > \Pi_A(\omega_A), \quad (22)$$

depends jointly on $(\omega_A, k, \alpha, \pi)$: the dip must be deep enough for a spread with weights $(1 - \pi, \pi)$ to beat the depressed target. Feasibility requires $\omega_W \geq \varepsilon$.

2.2.4 W's willingness to trade

At $q = \pi$, W is content with no trade. For $q > \pi$, the state-1 claim is overpriced relative to its probability, so selling it—delivering ε to A in state 1 and collecting $\frac{q}{1-q}\varepsilon$ from A in state 0—yields a positive expected return. Since u_W is strictly concave, W is risk-averse and requires this premium to bear the consumption risk across states.

W 's first-order condition (19) implicitly defines W 's optimal trade $\varepsilon_W(q)$. Since u_W is C^2 with $u_W'' < 0$, the implicit function theorem applies at $(\varepsilon, q) = (0, \pi)$. Differentiating (19) totally and evaluating at that point:

$$\varepsilon'_W(\pi) = \frac{-u'_W(\omega_W)}{\pi u''_W(\omega_W)} > 0, \quad (23)$$

since $u'_W > 0$ and $u''_W < 0$. So as q rises above π , W is willing to supply a positive ε —selling state-1 consumption to A in exchange for an actuarially favorable return in state 0.

2.2.5 Equilibrium existence

We now have a mismatch at $q = \pi$: country A wants to buy the state-1 claim ($\varepsilon > 0$), but W is content with no trade ($\varepsilon_W(\pi) = 0$). There is excess demand for the state-1 security at the fair price. As q rises above π , the state-1 claim becomes more expensive for A and more attractive for W to sell. The question is whether there exists a price $q^* > \pi$ at which both agents agree on a common $\varepsilon^* > 0$.

On W 's side, $\varepsilon_W(q)$ is well-defined and smooth for $q \in (\pi, 1)$, with $\varepsilon_W(\pi) = 0$ and $\varepsilon'_W(\pi) > 0$. On A 's side, the objective $F(\varepsilon; q)$ is continuous in (ε, q) , and for each q the feasible set is compact (bounded by $\varepsilon \leq \omega_W$ and $V_A(0) \geq 0$). By the maximum theorem, A 's optimal correspondence $\varepsilon_A(q)$ is upper hemicontinuous and A 's value $F^*(q) \equiv \max_\varepsilon F(\varepsilon; q)$ is continuous.

Define excess demand $z(q) = \varepsilon_A(q) - \varepsilon_W(q)$, taking the largest selection from ε_A where needed.

- At $q = \pi$: by (22), $F^*(\pi) > \Pi_A(\omega_A)$, so $\varepsilon_A(\pi) > 0$. Since $\varepsilon_W(\pi) = 0$, we have $z(\pi) > 0$.

- As $q \rightarrow 1$: the cost of each unit of ε in state 0 is $\frac{q}{1-q} \rightarrow \infty$, so A 's feasible trade shrinks to zero: $\varepsilon_A(q) \rightarrow 0$. Meanwhile $\varepsilon_W(q)$ remains positive. So $z(q) < 0$ for q close to 1.

Since z changes sign on $(\pi, 1)$, a zero exists provided z is continuous. The subtlety is that $\varepsilon_A(q)$ may jump: A 's non-concave objective can have multiple local maxima that exchange dominance as q varies. By the maximum theorem, A 's optimal set is an upper hemicontinuous correspondence, so such jumps are “inward” (the set of optima cannot suddenly expand). Generically in the parameters $(\omega_A, \omega_W, \pi)$, A 's optimum is unique for each q , making ε_A a continuous function and the intermediate value theorem applicable.

Proposition 2. *Fix $k \in (0, 1]$, $\alpha \in (0, 1)$, and $\omega_B > 0$. Let V_A^{war} be the war threshold from Proposition 1. Suppose:*

1. $\omega_A > V_A^{\text{war}}$;
2. *A gains from trade at fair odds: there exists a feasible $\varepsilon > 0$ (i.e., $\varepsilon \leq \omega_W$ and $\omega_A - \frac{\pi}{1-\pi}\varepsilon \geq 0$) such that $F(\varepsilon; \pi) > \Pi_A(\omega_A)$;*
3. *A 's optimum $\varepsilon_A(q)$ is unique for each $q \in (\pi, 1)$.*

Then there exists a nontrivial sunspot equilibrium: a price $q^ \in (\pi, 1)$ and a trade $\varepsilon^* > 0$ such that ε^* maximizes $F(\varepsilon; q^*)$, ε^* satisfies (19) at $q = q^*$, and markets clear.*

Proof. By (i) and Proposition 1(i)–(ii), $\Pi_A(\omega_A) < \omega_A^\alpha$. By (ii), $F(\varepsilon; \pi) > \Pi_A(\omega_A) = F(0; \pi)$ for some feasible $\varepsilon > 0$, so A 's optimal trade at $q = \pi$ satisfies $\varepsilon_A(\pi) > 0$, while $\varepsilon_W(\pi) = 0$. So $z(\pi) = \varepsilon_A(\pi) - \varepsilon_W(\pi) > 0$. As $q \rightarrow 1$, the feasible set $[0, \bar{\varepsilon}(q)]$ shrinks to $\{0\}$, so $\varepsilon_A(q) \rightarrow 0$, while $\varepsilon_W(q)$ remains positive. So $z(q) < 0$ for q sufficiently close to 1. By (iii), ε_A is a continuous function, and ε_W is smooth, so z is continuous on $(\pi, 1)$. By the intermediate value theorem, there exists $q^* \in (\pi, 1)$ with $z(q^*) = 0$, i.e., $\varepsilon_A(q^*) = \varepsilon_W(q^*) \equiv \varepsilon^* > 0$. At this (q^*, ε^*) , A is optimizing by definition of ε_A , W 's FOC holds by definition of ε_W , and market clearing holds since both agree on ε^* . \square

In the nontrivial equilibrium, $V_A(1) = \omega_A + \varepsilon^* > \omega_A > V_A^{\text{war}}$, so state 1 has conflict—tribute or war. For the sunspot to create a genuine peace-versus-conflict divide, we need $V_A(0) < V_A^{\text{war}}$, i.e.,

$$\omega_A - \frac{q^*}{1 - q^*} \varepsilon^* < V_A^{\text{war}}. \quad (24)$$

This holds whenever ε^* is large enough relative to $\omega_A - V_A^{\text{war}}$. If condition (ii) is verified with $V_A(0) < V_A^{\text{war}}$ at $q = \pi$ —that is, if A 's gain from trade at fair odds comes from crossing the war threshold—then $\varepsilon_A(\pi)$ places $V_A(0)$ below V_A^{war} . When ω_W is large, W is nearly risk-neutral, q^* is close to π , and A 's equilibrium trade ε^* is close to $\varepsilon_A(\pi)$. So (24) holds: the sunspot determines whether there is peace or conflict.

Corollary 3. *Fix $k \in (0, 1]$, $\alpha \in (0, 1)$, and $\omega_B > 0$. There exists an open interval $I \ni V_A^{\text{fight}}$ such that for every $\omega_A \in I$ and every ω_W sufficiently large, a nontrivial sunspot equilibrium exists.*

Proof. At $\omega_A = V_A^{\text{fight}}$ and $q = \pi$, the convex kink (Proposition 1(v)) gives A a first-order gain from trade:

$$F(\varepsilon; \pi) - \Pi_A(\omega_A) = \pi\varepsilon[\Pi'_A(V_A^{\text{fight}+}) - \Pi'_A(V_A^{\text{fight}-})] + O(\varepsilon^2) > 0 \quad (25)$$

for $\varepsilon > 0$ small. Since both one-sided derivatives of Π_A vary continuously in V_A , and $\Pi'_A(V_A^{\text{fight}+}) > \Pi'_A(V_A^{\text{fight}-})$ is a strict inequality, the gain from trade persists for ω_A in a neighborhood of V_A^{fight} by the implicit function theorem. This verifies condition (ii) of Proposition 2. For ω_W large, W is nearly risk-neutral, conditions (i) and (iii) hold, and the conclusion follows. \square

2.3 Numerical illustrations

The preceding results establish the existence of sunspot equilibria for endowments near V_A^{fight} . We now compute equilibria explicitly to illustrate the mechanism and its comparative statics. Throughout, the World has CRRA utility $u_W(x) = x^\beta$ with $\beta = 0.5$, endowment $\omega_W = 10,000$, and the sunspot probability is $\pi = 0.1$.

Structural parameters. Table 1 reports the tribute ratio r , kink ratio Ω/Z^* , and thresholds V_A^{war} and V_A^{fight} for several (α, k) combinations. The tribute ratio r is small (below 0.25 in all cases), confirming that B is the poorer player after transfer. The kink ratio $\Omega/Z^* = 1/(1 - k\tilde{\sigma}_0)$ ranges from 1.06 to 1.15: the jump in Π'_A at V_A^{fight} is between 6% and 15% of the left derivative. Both the tribute region $(V_A^{\text{war}}, V_A^{\text{fight}})$ and the kink size grow as α falls, reflecting the deeper “sag” in Π_A relative to V_A^α .

α	k	r	Ω/Z^*	V_A^{war}	V_A^{fight}
0.99	0.5	0.217	1.130	3.60	36.92
0.99	0.8	0.243	1.149	3.11	46.72
0.99	1.0	0.247	1.152	3.05	56.76
0.95	0.5	0.197	1.116	4.07	47.46
0.95	0.8	0.228	1.138	3.39	56.10
0.95	1.0	0.234	1.142	3.27	66.36
0.90	0.5	0.170	1.097	4.88	68.78
0.90	0.8	0.206	1.122	3.85	73.39
0.90	1.0	0.216	1.129	3.63	83.26

Table 1: Structural parameters of the tribute and war regions. The tribute ratio $r = (\omega_B + \bar{T})/\Omega$ is constant across V_A (Lemma 6). The kink ratio Ω/Z^* gives the factor by which Π'_A jumps at V_A^{fight} .

Computed equilibria. Table 2 reports sunspot equilibria in two panels. The top panel sets $\omega_A = V_A^{\text{fight}}$, placing A 's endowment exactly at the convex kink; A trades to shift $V_A(0)$ into the tribute region and $V_A(1)$ into the war region. The bottom panel sets ω_A below V_A^{fight} (in the tribute region) for α near one, where the equilibrium pushes $V_A(0)$ below V_A^{war} so that the sunspot determines genuine peace versus war. The equilibrium price q^* exceeds the sunspot probability $\pi = 0.1$, reflecting A 's willingness to pay a premium for the high-state claim. A 's gain ranges from 1% to 15% of the no-trade payoff. The gains are large because ω_W is large (W is nearly risk-neutral), so the equilibrium price q^* is close to π and A captures most of the surplus. With a smaller ω_W , A 's gains would be smaller but still positive (Corollary 3).

3 Discussion

The model identifies a specific channel through which financial market access can destabilize an otherwise peaceful relationship. A country that is paying tribute to maintain peace has a non-concave reduced-form payoff; if it can trade state-contingent claims with an outside party, it will exploit the non-concavity by randomizing over tribute and war. The mechanism is purely structural: it requires no private information, no behavioral departures, and no shocks to fundamentals. What it does require is a financial market—an

<i>Tribute \leftrightarrow war equilibria</i>							
α	k	ω_A	q^*	ε^*	$V_A(0)$	$V_A(1)$	regime(0)/regime(1)
0.99	0.5	36.9	0.102	295	3.6	332	tribute/war
0.95	0.5	47.5	0.102	295	14.1	343	tribute/war
0.95	1.0	66.4	0.103	475	12.2	541	tribute/war
0.90	1.0	83.3	0.101	207	60.0	290	tribute/war
<i>Peace \leftrightarrow war equilibria</i>							
α	k	ω_A	q^*	ε^*	$V_A(0)$	$V_A(1)$	regime(0)/regime(1)
0.99	0.5	31.9	0.101	251	3.6	283	peace/war
0.99	0.5	38.9	0.102	312	3.6	351	peace/war
0.99	1.0	54.8	0.102	454	3.1	508	peace/war

Table 2: Computed sunspot equilibria ($\omega_B = 1$, $\pi = 0.1$, $u_W(x) = x^{0.5}$, $\omega_W = 10,000$). Top panel: $\omega_A = V_A^{\text{fight}}$, so A 's endowment sits at the convex kink in Π_A . Bottom panel: ω_A in the tribute region; for α near one, the equilibrium drives $V_A(0)$ below V_A^{war} , so state 0 has genuine peace (no military tension).

investor willing to take the other side of the trade.

This connects to a growing empirical literature on the relationship between financial integration and conflict, but with a twist. [Gartzke \[2007\]](#) argues that capital market integration is pacifying: markets reveal information about resolve, raise the opportunity cost of fighting, and help states settle disputes short of war. The “capitalist peace” displaces the democratic peace once financial openness is controlled for. The model in this paper identifies a channel running in the opposite direction. The same financial infrastructure that Gartzke credits with promoting peace—state-contingent claims, liquid capital markets, willing counterparties—is what enables country A to exploit the non-concavity created by tribute. The two channels are not contradictory: Gartzke’s operates through information and opportunity costs, the one here through the geometry of payoffs. But the latter implies that financial market completion is not unambiguously pacifying, even with complete information and fully rational agents.

The empirical evidence on financial liberalization and conflict is consistent with this ambiguity. [Bussmann and Schneider \[2007\]](#) find that the *level*

of economic openness reduces civil war risk, but the *process* of liberalizing increases it—a pattern they call the “J-curve” of reform. [Hartzell et al. \[2010\]](#) show that countries adopting IMF structural adjustment programs face significantly higher civil war onset, and that compensatory measures (social spending, foreign aid) do not offset the effect. Most directly, [Chapman and Reinhart \[2013\]](#) find that exogenous increases in sovereign bond yield spreads—a measure of the price of foreign capital—robustly predict civil conflict onset in developing countries. These papers attribute the destabilization to redistributive shocks or austerity: liberalization creates losers whose opportunity cost of fighting falls. The model here offers a complementary mechanism that operates at the interstate level and does not require domestic losers. The instability comes not from redistribution but from market completion: access to Arrow securities allows a country to trade into the non-concave region of its payoff, and the demand for randomization that results is structural, not a policy failure that transfers can remedy.

The distinction between *levels* and *processes* in the empirical literature maps onto a distinction in the model. A country’s endowment ω_A determines the level; access to the Arrow security market determines the process.

The Nagorno-Karabakh conflict between Azerbaijan and Armenia illustrates the pattern. After the 1994 ceasefire, Azerbaijan—the larger state—accepted costly concessions: loss of roughly thirteen percent of its internationally recognized territory and the displacement of hundreds of thousands of citizens. This is the tribute region of the model: the stronger country bearing a structural cost to maintain a fragile peace. What changed was not the territorial dispute but Azerbaijan’s financial capacity. Oil revenues from the Baku-Tbilisi-Ceyhan pipeline, a sovereign wealth fund accumulating tens of billions in assets, and access to international bond markets transformed Azerbaijan’s outside option. By 2015, Azerbaijan’s military budget exceeded Armenia’s entire national budget. The 2020 war had no new fundamental trigger—the same territory, the same dispute, the same parties. In the language of the model, the “sunspot” was not a shock to fundamentals but the realization of a state in which Azerbaijan had positioned itself to fight rather than continue paying tribute, enabled by the financial instruments that the international capital market supplied. A country can be wealthy enough that tribute is affordable, yet the availability of contingent-claims trading unlocks sunspot equilibria that would be impossible in autarky. The model thus provides a formal mechanism for the empirical finding that it is not openness per se but the *act of opening* that destabilizes.

4 Conclusion

The central claim of this paper is that the costs of maintaining peace under anarchy can reshape a state’s effective risk preferences. When a risk-averse country must pay tribute to deter a potential aggressor, the tribute distorts its reduced-form payoff over wealth, creating a convex kink at the boundary where the country switches from paying tribute to fighting. Over a range of endowments near this boundary, a country that is fundamentally risk-averse behaves as if it were risk-accepting. When the country can trade state-contingent claims with an outside investor, a sunspot equilibrium exists: an extrinsic random variable, unrelated to any fundamental, determines whether the country pays tribute or fights.

The paper’s contribution to the costly peace literature is to show that the costs identified by [Fearon \[2018\]](#) and [Coe \[2011\]](#) are not merely wasteful—they are distortionary. Tribute reshapes the functional relationship between a country’s endowment and its welfare, creating a demand for volatility that financial markets can supply. In this sense, anarchy does not just impose costs—it generates instability.

Several extensions are natural. The model excludes country B from the financial market; endogenizing B ’s participation would complicate the first-stage analysis but would not eliminate the non-concavity that drives the result. The model is static; in a repeated setting, the dynamics of tribute, militarization, and financial positions would interact in ways the current framework cannot capture. Whether the demand for volatility persists, amplifies, or is disciplined by repeated interaction is an open question. A related question is whether the logic extends to monetary integration, where joining a currency union eliminates some state-contingent instruments (independent monetary policy, devaluation) while deepening access to others (sovereign bond markets, cross-border capital flows). The interaction between the commitment embedded in monetary union and the demand for volatility identified here is a natural setting for a dynamic extension. The broader implication is that the structure of international bargaining can generate demand for financial instruments that tie a country’s resources to extrinsic events. The sunspot need not be literally a sunspot—any extrinsic variable on which contracts can be written will do. What matters is that the costs of peace create a non-concavity, and the non-concavity creates a demand for randomization that financial markets can supply.

References

- Carmen Beviá and Luis C. Corchón. Peace agreements without commitment. *Games and Economic Behavior*, 68(2):469–487, 2010. doi: 10.1016/j.geb.2009.09.012.
- Margit Bussmann and Gerald Schneider. When globalization discontent turns violent: Foreign economic liberalization and internal war. *International Studies Quarterly*, 51(1):79–97, 2007. doi: 10.1111/j.1468-2478.2007.00440.x.
- David Cass and Karl Shell. Do sunspots matter? *Journal of Political Economy*, 91(2):193–227, 1983. doi: 10.1086/261139.
- Terrence Chapman and Eric Reinhardt. Global credit markets, political violence, and politically sustainable risk premia. *International Interactions*, 39(3):316–342, 2013. doi: 10.1080/03050629.2013.782302.
- Andrew J. Coe. Costly peace: A new rationalist explanation for war. Working paper, University of Southern California, 2011.
- James D. Fearon. Rationalist explanations for war. *International Organization*, 49(3):379–414, 1995. doi: 10.1017/S0020818300033324.
- James D. Fearon. Cooperation, conflict, and the costs of anarchy. *International Organization*, 72(3):523–559, 2018. doi: 10.1017/S0020818318000115.
- Michelle R. Garfinkel. Arming as a strategic investment in a cooperative equilibrium. *American Economic Review*, 80(1):50–68, 1990.
- Erik Gartzke. The capitalist peace. *American Journal of Political Science*, 51(1):166–191, 2007. doi: 10.1111/j.1540-5907.2007.00244.x.
- Hein E. Goemans and Mark Fey. Risky but rational: War as an institutionally induced gamble. *Journal of Politics*, 71(1):35–54, 2009. doi: 10.1017/S0022381608090038.
- Caroline A. Hartzell, Matthew Hoddie, and Molly Bauer. Economic liberalization via IMF structural adjustment: Sowing the seeds of civil war? *International Organization*, 64(2):339–356, 2010. doi: 10.1017/S0020818310000068.

- Adam Meirowitz, Kristopher W. Ramsay, and Noam Reich. Militarization, negotiations, and conflict. In *Handbook of the Economics of Conflict*, volume 1. Elsevier, 2024. doi: 10.1016/bs.hoec.2024.10.007.
- Robert Powell. Guns, butter, and anarchy. *American Political Science Review*, 87(1):115–132, 1993. doi: 10.2307/2938960.
- Robert Powell. War as a commitment problem. *International Organization*, 60(1):169–203, 2006. doi: 10.1017/S0020818306060061.
- Edward C. Prescott and Karl Shell. Introduction to sunspots and lotteries. *Journal of Economic Theory*, 107(1):1–10, 2002. doi: 10.1006/jeth.2002.2946.
- Karl Shell. Monnaie et allocation intertemporelle. *Séminaire d'Économétrie Roy-Malinvaud*, 1977. Paris, Centre National de la Recherche Scientifique.

A Proofs and Supplementary Material

A.1 Symmetric Regimes

We verify that neither country prefers war in either symmetric regime.

Both unconstrained ($e^\dagger \leq \min\{V_A, 1\}$). The equilibrium is (e^\dagger, e^\dagger) with war payoffs

$$\Psi_A = \Psi_B = \frac{u(Z^*)}{2}, \quad Z^* = (V_A + 1) \frac{\alpha}{1 + \alpha}. \quad (26)$$

Country i prefers peace iff $u(V_i) \geq \Psi_i$, equivalently $2^{1/\alpha} V_i \geq Z^*$. Since $V_i > e^\dagger$ in this regime,

$$2^{1/\alpha} V_i > \frac{2^{1/\alpha}}{2(1 + \alpha)k} \cdot (V_A + 1) = \frac{2^{1/\alpha}}{2\alpha k} \cdot Z^* \geq Z^*, \quad (27)$$

where the last inequality uses $2^{1/\alpha} \geq 2\alpha k$ (since $1/\alpha \geq 1$ and $\alpha k \leq 1$).

Both constrained ($e^\dagger \geq \max\{V_A, 1\}$). The equilibrium is $(V_A, 1)$ and

$$\Psi_i = \frac{V_i}{\Omega} u(\Omega(1 - k)). \quad (28)$$

Country i prefers peace iff $u(V_i) \geq \Psi_i$, which reduces to $1 \geq (V_i/\Omega)^{1-\alpha}(1 - k)^\alpha$ —and this holds since $V_i/\Omega \leq 1$ and $1 - k \leq 1$.

In both cases, $\Pi_A(V_A) = V_A^\alpha$.

A.2 Asymmetric Regime Derivations

FOC and the quadratic for s^* . In the asymmetric regime ($e_B^* = 1$, A unconstrained), A 's first-order condition is

$$\frac{1}{(e_A + 1)^2} u(Z) = \frac{e_A}{e_A + 1} k u'(Z), \quad Z \equiv V_A + 1 - k(e_A + 1). \quad (29)$$

Using $u(Z)/u'(Z) = Z/\alpha$, this simplifies to

$$\frac{Z}{\alpha e_A(e_A + 1)} = k. \quad (30)$$

Substituting $Z = V_A + 1 - k(e_A + 1)$ and letting $s = e_A + 1$:

$$\alpha k s^2 + (1 - \alpha)k s - (V_A + 1) = 0, \quad (31)$$

whose positive root is (8).

A prefers peace. Since A is unconstrained, $e_A^* < V_A$. By concavity of u (with $u(0) = 0$),

$$\Psi_A = \frac{e_A^*}{s^*} u(Z^*) \leq u\left(\frac{e_A^* Z^*}{s^*}\right). \quad (32)$$

But $e_A^*/s^* < V_A/(V_A + 1)$ (since $e_A^* < V_A$ and $s^* = e_A^* + 1$), and $Z^* < V_A + 1$, so

$$\frac{e_A^* Z^*}{s^*} < \frac{V_A}{V_A + 1} \cdot (V_A + 1) = V_A. \quad (33)$$

Since u is strictly increasing, $\Psi_A < u(V_A)$.

Tribute closed forms. When B prefers war and A offers a transfer $T \geq 0$, the contest at $(V_A - T, 1 + T)$ has total effort $s^*(T)$ equal to

$$\frac{-(1 - \alpha)k(1 + T) + \sqrt{(1 - \alpha)^2 k^2 (1 + T)^2 + 4\alpha k(1 + T)(V_A + 1)}}{2\alpha k}, \quad (34)$$

with $e_A^*(T) = s^*(T) - (1 + T)$, $e_B^*(T) = 1 + T$, and $Z^*(T) = V_A + 1 - k s^*(T)$. Country j 's war payoff as a function of T is

$$\Psi_j(T) = \frac{e_j^*(T)}{s^*(T)} u(Z^*(T)). \quad (35)$$

A.3 The War Game Has a Unique Equilibrium

We establish that for any $V_A > 0$, the second-stage game has a unique subgame perfect equilibrium. The argument follows [Beviá and Corchón \[2010\]](#); we reproduce it in our notation for completeness.

Lemma 4 (Contest Equilibrium). *For any post-transfer endowments $\hat{V}_A > 0$ and $\hat{V}_B > 0$, the contest has a unique Nash equilibrium (e_A^*, e_B^*) with $e_A^*, e_B^* > 0$. The equilibrium efforts and payoffs are smooth functions of $(\hat{V}_A, \hat{V}_B, k)$ within each regime (unconstrained, one constrained, both constrained).*

Proof. We first show that corner solutions are not equilibria, then establish uniqueness of the interior solution.

No corner equilibria. If $e_A = e_B = 0$, the tie-breaking rule gives each country payoff $\frac{1}{2}u(\Omega)$. But either can deviate to $e_i = \varepsilon > 0$ and win with certainty, earning $u(\Omega - k\varepsilon) > \frac{1}{2}u(\Omega)$ for ε small. If $e_A = 0$ and $e_B > 0$, A 's payoff is 0, but any $\varepsilon > 0$ yields a positive payoff. Symmetrically, $e_B = 0$ with $e_A > 0$ is not a best response for B .

Uniqueness (unconstrained). At an interior equilibrium, the first-order conditions are

$$\frac{\partial p_i}{\partial e_i} u(\Omega - kE) - p_i k u'(\Omega - kE) = 0, \quad i = A, B, \quad (36)$$

where $E = e_A + e_B$. Since A and B share u , dividing A 's FOC by B 's eliminates the utility terms, leaving an equation in the ratio $\rho = e_A/e_B$ alone. With the symmetric proportional CSF ($p_i = e_i/E$), this gives $\rho^* = 1$ when endowments are symmetric, and a unique ρ^* determined by the constraint structure otherwise. Log-concavity of each player's payoff in own effort (verified by computing $\partial^2 \log U_i / \partial e_i^2 < 0$) ensures uniqueness.

Constrained cases. If the interior optimum exceeds a player's endowment, that player is constrained at their endowment. The other player's best response is unique by log-concavity. In all cases, the equilibrium is unique and depends smoothly on parameters (by the implicit function theorem applied to the FOCs, or by continuity at the constraint boundary). \square

Lemma 5 (War Declaration and Transfer). *Fix $V_A > 0$. Let $\Psi_A(V_A, \omega_B)$ denote A 's expected war payoff at the contest equilibrium (with no transfer).*

- (i) *If $\Psi_A(V_A, \omega_B) \leq u(V_A)$, then A does not want war and no transfer occurs. $\Pi_A(V_A) = u(V_A)$.*
- (ii) *If $\Psi_A(V_A, \omega_B) > u(V_A)$ and a pacifying transfer exists, then B transfers $\hat{T}(V_A) > 0$ to A , where \hat{T} is the smallest value satisfying (11). $\Pi_A(V_A) = u(V_A + \hat{T}(V_A))$.*
- (iii) *If no feasible transfer pacifies A , war occurs and $\Pi_A(V_A) = \Psi_A(V_A, \omega_B)$.*

In cases (i) and (ii), Π_A is continuous and piecewise smooth in V_A .

Proof. Case (i) is immediate. For case (ii), the indifference condition $u(V_A + T) = \Psi_A(V_A + T, \omega_B - T)$ defines \hat{T} implicitly. Existence follows from the intermediate value theorem: at $T = 0$, A strictly prefers war; as T increases, A 's peace payoff rises while the war payoff changes continuously; at some point indifference is reached. Uniqueness and smoothness follow from the implicit function theorem, noting that the derivative of the indifference condition with respect to T is nonzero at the solution (the peace payoff is strictly increasing in T , while the war payoff responds differently due to the contest structure). For case (iii), $\Psi_A(V_A + T, \omega_B - T) > u(V_A + T)$ for all $T \in [0, \omega_B]$, so no transfer suffices. \square

A.4 Proof of Proposition 1 (Non-Concavity)

We first establish two derivative formulas, then prove the proposition.

Lemma 6 (Tribute homogeneity). *In the tribute region $V_A \in (V_A^{war}, V_A^{fight})$, the ratio*

$$r \equiv \frac{1 + \bar{T}(V_A)}{V_A + \omega_B} \quad (37)$$

is constant, depending only on (α, k) and not on V_A . Consequently:

1. $\bar{T}(V_A) = r(V_A + \omega_B) - \omega_B$, so $\bar{T}'(V_A) = r \in (0, \frac{1}{2})$;
2. A 's effective wealth is $V_A - \bar{T} = (1 - r)(V_A + \omega_B)$;
3. in the tribute region, $\Pi_A(V_A) = (1 - r)^\alpha (V_A + \omega_B)^\alpha$.

Proof. Write $\Omega = V_A + \omega_B$ for total resources (fixed through the transfer) and define scaled variables $\tilde{\sigma} = s^*/\Omega$ and $r = (1 + \bar{T})/\Omega = \hat{V}_B/\Omega$. In the asymmetric regime, B is constrained ($e_B = \hat{V}_B$) and A 's first-order condition gives the quadratic

$$\alpha k \tilde{\sigma}^2 + (1 - \alpha)k r \tilde{\sigma} - r = 0. \quad (38)$$

Post-contest resources are $Z^* = \Omega(1 - k\tilde{\sigma})$, and B 's war payoff is $\Psi_B = (r/\tilde{\sigma})\Omega^\alpha(1 - k\tilde{\sigma})^\alpha$. The indifference condition $\Psi_B = u(\hat{V}_B) = (r\Omega)^\alpha$ becomes

$$r^{\alpha-1} \tilde{\sigma} = (1 - k\tilde{\sigma})^\alpha. \quad (39)$$

The system (38)–(39) involves only $(\tilde{\sigma}, r, \alpha, k)$ — Ω has dropped out. Hence r and $\tilde{\sigma}$ are constants determined by (α, k) alone.

Since $\bar{T} = r\Omega - \omega_B = r(V_A + \omega_B) - \omega_B$, we get $\bar{T}' = r$. The bound $r < \frac{1}{2}$ follows from B being the constrained (poorer) player after the transfer: $\hat{V}_B = r\Omega < (1 - r)\Omega = \hat{V}_A$ requires $r < \frac{1}{2}$. \square

Lemma 7 (War payoff derivative). *In the war region $V_A > V_A^{\text{fight}}$, the derivative of A 's war payoff is*

$$\Psi'_A(V_A) = \frac{e_A^*}{s^*} \alpha Z^{*\alpha-1}. \quad (40)$$

Proof. We will appeal to the envelope theorem. Country A chooses e_A to maximize $U_A(e_A) = \frac{e_A}{e_A+1} u(V_A + 1 - k(e_A + 1))$. At the interior optimum e_A^* , the derivative of $\Psi_A = U_A(e_A^*)$ with respect to the parameter V_A equals the partial derivative of U_A with respect to V_A evaluated at $e_A = e_A^*$:

$$\Psi'_A(V_A) = \frac{\partial}{\partial V_A} \left[\frac{e_A}{e_A+1} u(V_A + 1 - k(e_A + 1)) \right]_{e_A=e_A^*} = \frac{e_A^*}{s^*} u'(Z^*), \quad (41)$$

which gives (40) since $u'(Z) = \alpha Z^{\alpha-1}$. \square

Proof of Proposition 1. Parts (i)–(ii) follow from the analysis in Section 2: in the symmetric regimes (Appendix A.1), $\Pi_A = V_A^\alpha$; in the asymmetric regime, $\Psi_A < u(V_A)$ (Appendix A.2), so any tribute or contest cost pushes Π_A below V_A^α . Part (iii): from (8), $s^* = O(\sqrt{V_A})$ as $V_A \rightarrow \infty$, so $Z^* = V_A + 1 - ks^* \sim V_A$ and $e_A^*/s^* \rightarrow 1$, giving $\Psi_A/V_A^\alpha \rightarrow 1$. Part (iv), continuity, follows from Lemma 4 (smooth dependence on parameters within each regime) and the fact that the transition conditions are equalities.

For part (v), we show a convex kink at V_A^{fight} . Write $\Omega = V_A^{\text{fight}} + \omega_B$ for total resources.

Left derivative. By Lemma 6, in the tribute region $\Pi_A = (1 - r)^\alpha \Omega^\alpha$, so

$$\Pi'_A(V_A^{\text{fight}-}) = \alpha (1 - r)^\alpha \Omega^{\alpha-1}. \quad (42)$$

Right derivative. By Lemma 7, $\Psi'_A = (e_A^*/s^*) \alpha Z^{*\alpha-1}$.

Ratio. At V_A^{fight} , A 's indifference gives $(1 - r)^\alpha \Omega^\alpha = (e_A^*/s^*) Z^{*\alpha}$, so

$$\frac{e_A^*}{s^*} = \frac{(1 - r)^\alpha \Omega^\alpha}{Z^{*\alpha}} \quad (43)$$

and therefore

$$\Pi'_A(V_A^{\text{fight}+}) = \frac{(1-r)^\alpha \Omega^\alpha}{Z^{*\alpha}} \cdot \alpha Z^{*\alpha-1} = \alpha (1-r)^\alpha \Omega^{\alpha-1} \cdot \frac{\Omega}{Z^*}. \quad (44)$$

Comparing (42) and (44):

$$\frac{\Pi'_A(V_A^{\text{fight}+})}{\Pi'_A(V_A^{\text{fight}-})} = \frac{\Omega}{Z^*} = \frac{1}{1 - k\tilde{\sigma}_0} > 1, \quad (45)$$

where $\tilde{\sigma}_0 = s^*/\Omega > 0$ is the effort-to-resource ratio in the (no-tribute) contest. The inequality holds trivially: $k > 0$ and $\tilde{\sigma}_0 > 0$ (both countries exert positive effort), so $k\tilde{\sigma}_0 > 0$. \square

A.5 Proof of Proposition 2 (Sunspot Equilibrium)

Proof. The proof in the main text establishes the result via the intermediate value theorem applied to excess demand $z(q) = \varepsilon_A(q) - \varepsilon_W(q)$. Here we provide an alternative argument using the convex kink at V_A^{fight} .

Fix $\omega_A = V_A^{\text{fight}}$ and consider the no-trade allocation at $q = \pi$. Let $\varepsilon > 0$ be small, so that $V_A(0) = \omega_A - \frac{\pi}{1-\pi}\varepsilon$ (tribute region) and $V_A(1) = \omega_A + \varepsilon$ (war region).

A 's gain from this deviation is

$$\begin{aligned} \Delta U_A &= (1-\pi) [\Pi_A(V_A(0)) - \Pi_A(\omega_A)] + \pi [\Pi_A(V_A(1)) - \Pi_A(\omega_A)] \\ &= \pi\varepsilon [\Pi'_A(V_A^{\text{fight}+}) - \Pi'_A(V_A^{\text{fight}-})] + O(\varepsilon^2) > 0, \end{aligned} \quad (46)$$

where the leading term is positive by the convex kink (Proposition 1(v)).

For W , the allocation changes from ω_W in both states to $\omega_W + \frac{\pi}{1-\pi}\varepsilon$ in state 0 and $\omega_W - \varepsilon$ in state 1. By strict concavity of u_W , W 's loss is $|\Delta U_W| = O(\varepsilon^2)$.

Since A 's gain is $O(\varepsilon)$ and W 's loss is $O(\varepsilon^2)$, for ε small enough there exist gains from trade. Adjusting q above π to equate marginal rates of substitution yields an equilibrium with $\varepsilon^* > 0$. \square

A.6 A's Gain from Trade: Detailed Argument

The non-concavity of Π_A implies that there exist $V_A^L < \omega_A < V_A^R$ and a weight $\lambda \in (0, 1)$ with $\lambda V_A^L + (1-\lambda)V_A^R = \omega_A$ such that

$$\lambda \Pi_A(V_A^L) + (1-\lambda) \Pi_A(V_A^R) > \Pi_A(\omega_A). \quad (47)$$

At $q = \pi$, choosing ε so that $V_A(1) = V_A^R$ and $V_A(0) = V_A^L$ gives

$$F(\varepsilon; \pi) = (1 - \pi) \Pi_A(V_A^L) + \pi \Pi_A(V_A^R). \quad (48)$$

For this to equal the chord above, we would need $(1 - \pi) = \lambda$, but π is fixed.

The way around this is to use the price q to adjust the effective weights. At a general price q , the budget constraint gives

$$V_A(0) = \omega_A - \frac{q}{1 - q} \varepsilon, \quad V_A(1) = \omega_A + \varepsilon, \quad (49)$$

so A 's expected payoff at (ε, q) is a weighted average of Π_A evaluated at two points whose q -weighted mean is ω_A . By choosing q and ε jointly, A can place $V_A(0)$ and $V_A(1)$ anywhere on either side of ω_A , with the probability weights $(1 - \pi, \pi)$ fixed by nature.

At $q = \pi$, we choose ε so that $V_A(0)$ sits just below V_A^{war} , where we have $\Pi_A(V_A(0)) = V_A(0)^\alpha$. The corresponding $V_A(1) = (\omega_A - (1 - \pi)V_A(0))/\pi$. Since ω_A is in the dip, $\Pi_A(\omega_A) < \omega_A^\alpha$ —the target is depressed. Whether

$$F(\varepsilon; \pi) = (1 - \pi) V_A(0)^\alpha + \pi \Pi_A(V_A(1)) > \Pi_A(\omega_A) \quad (50)$$

holds depends on the depth of the dip at ω_A relative to π and α : the dip must be deep enough that the depressed target can be beaten by a mean-preserving spread with weights $(1 - \pi, \pi)$. This is a joint condition on $(\omega_A, k, \alpha, \pi)$.