

Preface

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1 What this is

These handouts cover the mathematics a graduate student in political science or political economy needs in order to read formal theory and to write it. The model is Kim Border’s notes for economists: small, focused documents on individual topics, with definitions, examples, and proofs, written so that each can be read independently and referenced as needed. The difference is the audience and the examples: these notes are written for political scientists, and the examples are drawn from across political science and political economy rather than economics.

2 On the language we use

There is a circularity at the bottom of any introduction to mathematical foundations. To present logic carefully, we need to talk about sets of formulas, functions on them, and rules for building them up. But to present set theory carefully, we need to use logical reasoning—definitions, “for all,” “there exists,” and so on. Whichever foundation we introduce first borrows informally from the other.

We resolve this in the standard way: by treating *informal mathematical English* as the metalanguage. Throughout the handouts, words like “set,” “function,” “list,” “rule,” “for every,” and “there exists” are used in the sense a working mathematician would—their meaning understood in advance, before any formal handout on naive set theory or first-order logic. When a later handout treats one of these notions formally, that formal treatment supersedes the informal usage *for arguments made inside that framework*, but it does not retroactively forbid the informal usage that appears elsewhere.

This is not circular. We never cite a formalized result before it has been formalized. We just borrow ordinary mathematical vocabulary, the way every textbook does.

3 Style and conventions

Each handout is self-contained, with its own definitions, theorems, examples, and (where useful) exercises. Notation is unified across the project: the symbols defined in the shared notation file mean the same thing in every handout. Examples are drawn from political science and political economy whenever a natural example exists; where the mathematics itself is the point, we use generic mathematical examples instead of forcing a political reading.

Theorem environments share a single counter, so a handout reads as Definition 1, Proposition 2, Lemma 3, and so on, regardless of type. References cited at the end of a handout are pulled from a master bibliography shared by the whole project.

4 What is assumed

Algebra and a year of calculus, at the level any first-year graduate student in political science will have. Other than that, nothing: the handouts build the rest from the ground up, in an order that minimizes—without eliminating—cross-reference.